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Determination of da/dN–ΔK_{I} curves for small cracks in alumina in alternating bending tests

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Crack-growth relations under cyclic fatigue conditions are mostly determined for long cracks. In order to determine da/dN–ΔK curves for small cracks from lifetimes under cyclic load a procedure has been derived which is based on a method usually applied to subcritical crack growth. To prove the cyclic effect and to demonstrate the procedure in detail, measurements were carried out on an Al_2O_3-ceramic in bending with an R-ratio of $R = -1$ and two types of relatively small cracks, namely natural cracks and Knoop-cracks. It was found that both crack types exhibit the same da/dN–ΔK relation. The exponent of the Paris law for fatigue crack growth is significantly different from the exponent of the power law for subcritical crack growth.

1. Introduction

In recent years much effort has been spent on the investigation of the cyclic fatigue behaviour of ceramic materials. It is obvious that in most cases damage due to fatigue starts at small flaws such as pores, inclusions or cracks. The fatigue damage, therefore, can be described applying linear elastic fracture mechanics by a relation between the crack growth rate per load cycle da/dN and the cyclic stress intensity factor ΔK. In metallic materials the da/dN–ΔK relation is determined with fracture mechanics specimens having a crack size of several centimetres and the results are applied to the real cracks in a component applying the appropriate stress intensity factor.

In ceramic materials, the behaviour of specimens with macrocracks was also investigated [1, 2]. The application of the results from macrocracks to components with small, natural flaws may be doubtful for several reasons: applicability of linear-elastic fracture mechanics to the small flaws, complicated flaw geometry, effects of rising crack growth resistance curve (R-curve). For crack extension under static loading large differences between micro- and macrocrack behaviour was observed [3]. A similar effect can be expected for cyclic loading.

The results of tests under cyclic loading can be presented as σ–N curves, with the stress amplitude, σ_a, or the maximum stress, σ_{max}, plotted versus the number of cycles until failure. It is also possible to derive from such tests the da/dN–ΔK relation of the material, as will be shown in this paper. This relation then can be compared with results from macrocracks.

The delayed failure under constant loading can be described by a relation between crack growth rate and stress intensity factor. From these results predictions for the cyclic fatigue behaviour can be made under the assumption that no additional fatigue damage occurs. The real fatigue can then be compared with the prediction to reveal any cyclic effects.

2. Cyclic fatigue relations

The crack growth in one load cycle da/dN is a function of the range of the stress intensity factor

$$\frac{da}{dN} = f(\Delta K) \tag{1}$$

where ΔK is dependent on the stress range, $\Delta\sigma$, and the crack depth, a, by

$$\Delta K = \Delta\sigma a^{1/2} Y \tag{2a}$$

Instead of the stress range the stress amplitude $\sigma_a = \Delta\sigma/2$ or the maximum stress can be used. Then with $R = \sigma_{min}/\sigma_{max}$

$$\Delta K = 2\sigma_a a^{1/2} Y \tag{2b}$$

$$\Delta K = \sigma_{max}(1 - R)a^{1/2} Y \tag{2c}$$

For a power law relation

$$\frac{da}{dN} = A(\Delta K)^n \tag{3a}$$

or

$$\frac{da}{dN} = A^*\left(\frac{\Delta K}{K_{Ic}}\right)^n \tag{3b}$$

the number of cycles until failure can be obtained by the integration of Equation 1

$$N_f = \frac{2}{A(n-2)Y^n(\Delta\sigma)^n}\left[\frac{1}{a_i^{(n-2)/2}} - \frac{1}{a_c^{(n-2)/2}}\right] \tag{4}$$

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where a_i is the initial crack depth and a_c is the critical crack depth. It is assumed that Y is independent of the crack depth. The parameter A or A^* and possibly also n may be dependent on R. Under the assumption that the inert fracture strength, σ_c, and cyclic fatigue are caused by the same flaw population, the initial flaw size can be replaced by the strength, σ_c

$$a_i = \left(\frac{K_{Ic}}{\sigma_c Y}\right)^2 \quad (5)$$

The critical flaw size is given by

$$a_c = \left(\frac{K_{Ic}}{\sigma_{max} Y}\right)^2 \quad (6)$$

leading to

$$N_f = \frac{2\sigma_c^{n-2}}{A(n-2)Y^n K_{Ic}^{n-2}(\Delta\sigma)^n}\left[1 - \left(\frac{\sigma_{max}}{\sigma_c}\right)^{n-2}\right] \quad (7)$$

Because of the large exponent n

$$\left(\frac{\sigma_{max}}{\sigma_c}\right)^{n-2} \ll 1 \quad (8)$$

and thus

$$N_f = \frac{B\sigma_c^{n-2}}{(\Delta\sigma)^n} = \frac{B\sigma_c^{n-2}}{\sigma_{max}^n(1-R)^n} = \frac{C}{\sigma_{max}^n(1-R)^n} \quad (9)$$

with

$$B = \frac{2}{A(n-2)Y^2 K_{Ic}^{n-2}} \quad (10a)$$

and

$$C = B\sigma_c^{n-2} \quad (10b)$$

In a plot of log $\Delta\sigma$ or log σ_{max} versus log N_f, a straight line with a slope of $-1/n$ is expected. Sometimes the stress is plotted versus the lifetime $t_f = N_f/f$, where f is the frequency. The scatter in lifetime can be related to the scatter in strength under the same assumptions: power law relation (Equation 3) and same flaw population for strength and lifetime. The scatter in strength is described by a Weibull distribution.

$$F(\sigma_c) = 1 - \exp\left[-\left(\frac{\sigma_c}{\sigma_0}\right)^m\right] \quad (11)$$

The scatter in the number of cycles to failure is then given by

$$F(N_f) = 1 - \exp\left[-\left(\frac{N_f}{N_0}\right)^{m^*}\right] \quad (12)$$

with

$$m^* = \frac{m}{n-2} \quad (13a)$$

$$N_0 = \frac{B\sigma_0^{n-2}}{(\Delta\sigma)^n} = \frac{B\sigma_0^{n-2}}{\sigma_{max}^n(1-R)^n} \quad (13b)$$

Equation (13b) is only valid if the strength and lifetime is determined with the same size of the specimen. The size effect in the Weibull statistics is described by the effective volume for volume flaws or the effective surface for surface flaws. To obtain the effective volume, V_{eff}, the stress distribution in the component is described as

$$\sigma(x, y, z) = \sigma^* g(x, y, z) \quad (14)$$

where σ^* is a reference stress and $g(x, y, z)$ a geometric function. Then V_{eff} is given by

$$V_{eff} = \int g^m(x, y, z)\mathrm{d}V \quad (15)$$

For different effective volumes considered in strength and lifetime determinations $V_{eff,\sigma}$ and $V_{eff,N}$, Equation 13b has to be replaced by

$$N_0 = \left(\frac{V_{eff,\sigma}}{V_{eff,N}}\right)^{1/m}\frac{B\sigma_0^{n-2}}{\Delta\sigma^n} \quad (16)$$

3. Prediction of cyclic fatigue from static tests

Under static loading the crack growth rate is a function of the stress intensity factor, K_I, which in many cases can be described by a power law relation

$$v = \begin{cases} A_s K_I^{n_s} & \text{for } K_I > 0 \\ 0 & \text{for } K_I \le 0 \end{cases} \quad (17)$$

A prediction of the lifetime under cyclic loading or of the crack growth $\mathrm{d}a/\mathrm{d}N$–ΔK curve can be made for any stress–time relation under the assumption that no additional effects are occurring. For a sinusoidal stress

$$\sigma(t) = \sigma_m + \sigma_a \sin\left(\frac{2\pi}{T}t\right) \quad (18)$$

with mean stress, σ_m the lifetime is obtained by integration of Equation 17, leading to

$$t_{fc} = \frac{N_f}{f} = \frac{B_s\sigma_c^{n_s-2}}{\sigma_a^{n_s}h(\sigma_m/\sigma_a, n_s)} \quad (19a)$$

$$t_{fc} = \frac{N_f}{f} = \frac{B_s\sigma_c^{n_s-2}}{\sigma_{max}^{n_s}h(\sigma_m/\sigma_a, n_s)}\left(\frac{2}{1-R}\right)^{n_s} \quad (19b)$$

with

$$h\left(\frac{\sigma_m}{\sigma_a}, n\right) = \frac{1}{\pi}\int_{-\alpha}^{\pi/2}[\sigma_m/\sigma_a + \sin\varphi]^n \mathrm{d}\varphi \quad (20a)$$

$$\alpha = \sin^{-1}(\sigma_m/\sigma_a) \quad (20b)$$

B_s is given by Equation 10, with A replaced by A_s.

The lifetime for static tests at the stress, σ_s is

$$t_{fs} = \frac{B_s\sigma_c^{n_s-2}}{\sigma_s^n} \quad (21)$$

Therefore, the lifetime under static load at the stress, σ_s can be compared to the lifetime under cyclic load according to

$$t_{fc} = t_{fs}\left(\frac{\sigma_s}{\sigma_{max}}\right)^n\left(\frac{2}{1-R}\right)^n\frac{1}{h(\sigma_m/\sigma_a, n)} \quad (22)$$

Equation (22) is valid under the assumption that the specimen sizes are the same for static and cyclic tests.

If static tests are performed in four-point bending and cyclic tests with the same specimen size in reversed bending ($R = -1$), then the effective volume (or surface) in the cyclic tests is twice that in the static tests. Then Equation 22 has to be replaced by

$$t_{fc} = t_{fs}\left(\frac{\sigma_s}{\sigma_{max}}\right)^n\left(\frac{2}{1-R}\right)^n\frac{1}{h(\sigma_m/\sigma_a, n)}\frac{1}{2^{(n-2)/m}} \quad (23)$$

For tests with $R = -1$, the function h is

$$h(0,n) = \frac{1}{2\pi^{1/2}}\left[\Gamma\left(\frac{n}{2}+\frac{1}{2}\right)\right]\bigg/\left[\Gamma\left(\frac{n}{2}+1\right)\right] \tag{24a}$$

which can be approximated for $n > 15$ by

$$h(0,n) \simeq \frac{0.395}{n^{1/2}} \tag{24b}$$

4. Evaluation of fatigue data with specimens containing natural cracks

The evaluation of tests with constant amplitude leads to a relation between the stress amplitude (or σ_{max}), lifetime (expressed as N_f or t_{fc}), and failure probability or to a relation between da/dN and ΔK.

4.1. Power law relation

For a power law relation according to Equation 3, the materials parameters A and n have to be determined.

In a plot of $\log \sigma_a$ (or $\log \sigma_{max}$) versus $\log N_f$ (or $\log t_f$) for at least two amplitudes, n and the quantity $C = B\sigma_c^{n-2}$ can be determined from a straight line through the median values of N_f. The parameter A then can be calculated from

$$A = \frac{2\sigma_c^{n-2}}{(n-2)Y^2 K_{Ic}^{n-2} C} \tag{25a}$$

or

$$A^* = \frac{2\sigma_c^{n-2} K_{Ic}^2}{(n-2)Y^2 C} \tag{25b}$$

where σ_c is the median inert strength. The factor Y depends on the flaw geometry. A value of $Y = 1.3$ for surface cracks is recommended.

4.2. Statistical procedure

A statistical procedure to determine the da/dN–ΔK relation is described here. It is analogous to the method developed by the authors for static tests [4]. It does not assume a power law relation between da/dN and ΔK. The evaluation procedure uses the general relation

$$N_f = \int_{a_i}^{a_c} \frac{da}{(da/dN)} = \frac{2}{(\Delta\sigma Y)^2}\int_{\Delta K_i}^{\Delta K_c} \frac{\Delta K\, d(\Delta K)}{(da/dN)} \tag{26}$$

with

$$\Delta K_i = \Delta\sigma Y a_i^{1/2} \tag{27}$$

$$\Delta K_c = \Delta\sigma Y a_c^{1/2} = \Delta\sigma_c Y a_i^{1/2} \tag{28}$$

and

$$\Delta\sigma_c = \sigma_c(1-R) \tag{29}$$

This first kind of Volterra integral equation can be solved by differentiation and one obtains

$$\frac{d(N_f \Delta\sigma^2 Y^2)}{d(\Delta K_i)} = -2\frac{\Delta K_i}{(da/dN)_i} \tag{30}$$

Assuming that the same flaws are responsible for fatigue failure and for strength, the initial crack size can be replaced by the inert fracture strength (Equation 5) and ΔK_i by

$$\Delta K_i = \frac{\Delta\sigma}{\sigma_c} K_{Ic} \tag{31}$$

Equation (30) then leads to the crack growth rate da/dN at the beginning of the cyclic test ($\Delta K = \Delta K_i$)

$$\left(\frac{da}{dN_i}\right) = -\frac{2K_{Ic}^2}{N_f \sigma_c^2 Y^2}\frac{d\log(\Delta\sigma/\sigma_c)}{d\log(N_f\Delta\sigma^2)} \tag{32}$$

If the initial crack size can be measured, Equation 32 can be replaced by

$$\left(\frac{da}{dN_i}\right) = -\frac{2a_i}{N_f}\frac{d\log(\Delta\sigma a_i^{1/2} Y_i)}{d\log(N_f \Delta\sigma^2 Y_i^2)} \tag{33}$$

The evaluation of Equation 32 requires the relation between $\Delta\sigma/\sigma_c$ and $N_f\Delta\sigma^2$. The basic idea is to use the natural scatter of the strength and the lifetime and to correlate the corresponding values for the same failure probability.

If for both the strength test and the lifetime test the same specimen size is used, then a number of N specimens are used for strength and lifetime measurements. The results are ranked and corresponding values (jth strength and jth lifetime) are used in a $\log(\Delta\sigma/\sigma_{cj})$ versus $\log(N_{fj}\Delta\sigma^2)$ plot. From this plot the slope is determined and da/dN calculated for the corresponding N_{fj}, σ_{cj} according to Equation 32. The corresponding ΔK_{ij} is given by

$$\Delta K_{ij} = \frac{\Delta\sigma}{\sigma_{cj}} K_{Ic} \tag{34}$$

If different numbers of specimens are used for lifetime and strength measurements, the corresponding values have to be obtained for the same failure probability according to

$$F_j = \frac{j-0.5}{N} \tag{35}$$

In principle, this method requires tests to be performed at only one stress range.

4.3. Description of a least-squares procedure

If the lifetime tests are performed at several stress levels and the numbers of results at each level are too small, it is recommended to apply a least-squares routine to find the mean dependency between $\Delta\sigma/\sigma_c$ and $\Delta\sigma^2 N_f$. If the fitting curve – not necessarily a straight line – is analytically expressed by

$$\log(\Delta\sigma^2 N_f) = f\left[\log\left(\frac{\Delta\sigma}{\sigma_c}\right)\right] \tag{36}$$

this relation can be introduced into Equation 30 which can be rewritten in a more convenient form which only contains the quantities $\Delta\sigma/\sigma_c$ and $\Delta\sigma^2 N_f$

$$\frac{da}{dN} = -\frac{2K_{Ic}^2}{Y^2}\left(\frac{\Delta\sigma}{\sigma_c}\right)^2 \frac{1}{\Delta\sigma^2 N_f} \left[d\left(\log\frac{\Delta\sigma}{\sigma_c}\right)\right]\bigg/[d(\log \Delta\sigma^2 N_f)] \tag{37}$$

Thus, the v–K curve is obtained directly as a mean curve. This variant seems to be best appropriate for artificial surface cracks with their strongly reduced scatter.

5. Testing device

Fig. 1 shows a simple testing device for cyclic bending tests with $R = -1$ [5]. The specimen (1) is fixed at its ends in brass tubes (2) by an epoxy resin (3). Owing to the low Young's modulus of the epoxy resin compared with the Young's modulus of the ceramic specimen, the load can be applied without any notch effect. The second version of fixing the specimen [6] may be recommended for materials with a relatively low Young's modulus (i.e. glass). The cyclic load is generated by the magnet system of a loudspeaker (4) and transferred to the specimen by a cantilever (5). The bending moment is measured by strain gauges (6) provided on the fixing bracket as well as directly on special specimens used for calibrating. At the moment of failure, the loudspeaker stops working and a time counter is interrupted. By additional application of a spring (7) the mean stress can be changed in a wide range.

6. Results from specimens with natural cracks

6.1. Strength and lifetime measurements

Measurements were carried out on 3.5 mm × 4.5 mm × 50 mm specimens made of 99.6% Al_2O_3. The 99.6% Al_2O_3 was roughly ground only resulting in a relatively low strength. Such a surface state will ensure that all specimens will fail on account of surface cracks. The specimens were annealed in vacuum for 5 h at 1200 °C. Fig. 2 shows in a Weibull plot inert bending strength data for the specimens with natural crack population. The corresponding strength for a specimen with twice the surface (or volume) σ_2, which

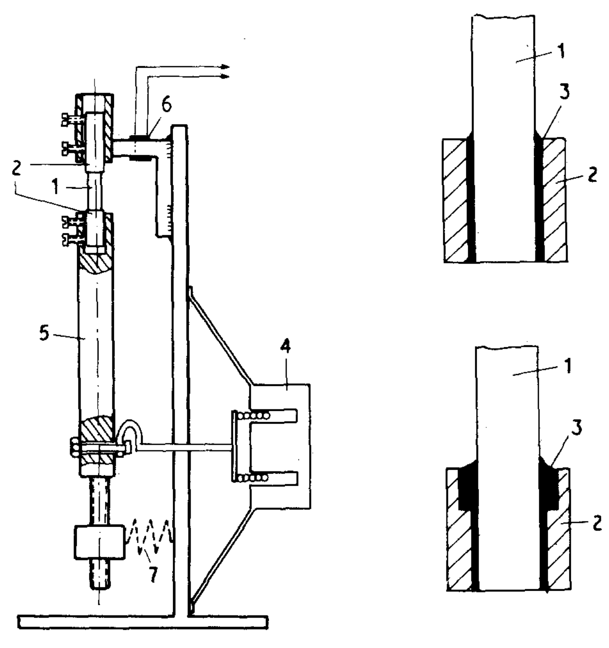

Figure 1 Testing device for alternating bending tests.

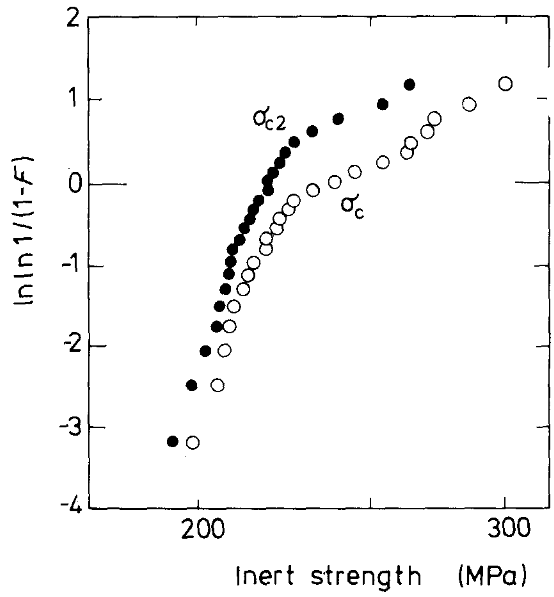

Figure 2 Inert strength in a Weibull representation.

is relevant for comparison with cyclic tests with $R = -1$, can be calculated by

$$F(\sigma_{c2}) = 1 - [1 - F(\sigma_c)]^2 \qquad (38)$$

The distribution of these strength data is also shown in Fig. 2. Obviously the data cannot be described by a two-parameter Weibull distribution. For further evaluation the following parameters have been determined:

median strength $\hat{\sigma}_c = 223$ MPa, $\hat{\sigma}_{c2} = 214$ MPa.

Weibull parameters for the data at the lower strength values:

$$\sigma_c\text{:}\quad F < 0.6,\quad \sigma_0 = 228.6\ \text{MPa},\quad m = 18.8;$$

$$\sigma_{2c}\text{:}\quad F < 0.72,\quad \sigma_0 = 220.1\ \text{MPa},\quad m = 21.1.$$

The results of tests with constant stress are shown in Fig. 3. From these data $n_s = 40$ was obtained. In Fig. 4 the results for $\sigma = 185$ MPa are shown in a Weibull plot. A similar behaviour as for the strength can be seen.

The Weibull parameters for $F < 0.6$ are

$$m_s^* = 0.616, \quad t_{s0} = 0.0194\ \text{h}.$$

From the scatter of strength and lifetime the relation between crack growth rate and stress intensity factor was found as [7]

$$\frac{da}{dt} = 5 \times 10^{-4}\left(\frac{K_I}{K_{Ic}}\right)^{38.6} \text{m s}^{-1} \qquad (39)$$

i.e. $\quad n = 38.6, \quad A_s^* = 5 \times 10^{-4}.$

In Fig. 5 the results of the cyclic tests are shown for a frequency of 50 Hz. Fig. 5 also shows the prediction from the static tests using Equation 23. The predicted and measured lifetimes are significantly different. It has to be concluded from these results that the cyclic fatigue effect is very strong.

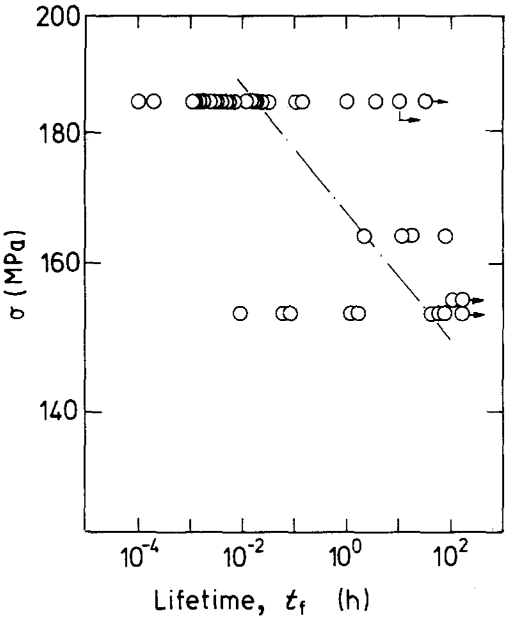

Figure 3 Constant load lifetime tests.

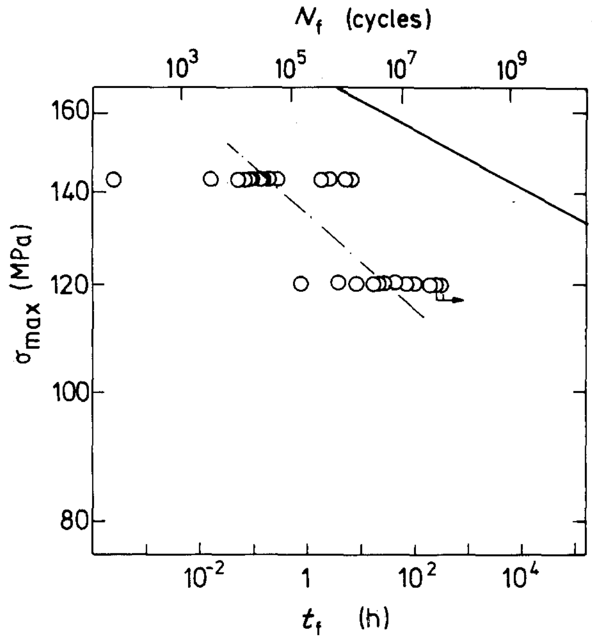

Figure 5 Cyclic lifetimes for specimens with natural cracks compared with predictions (——) based on static tests. $R = -1$; 50 Hz.

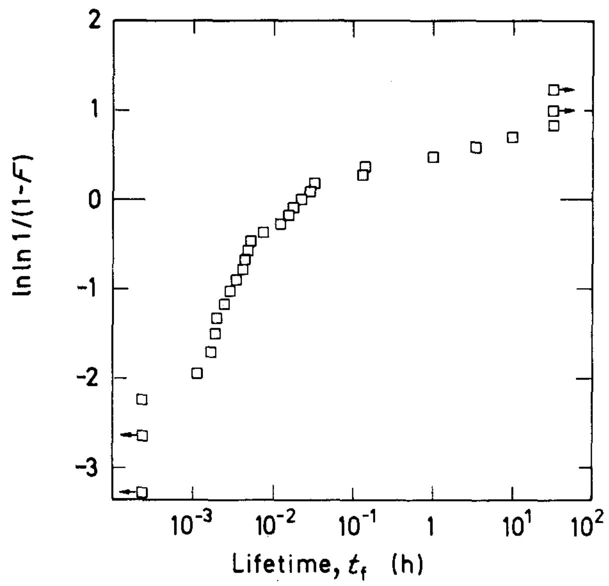

Figure 4 Static lifetimes in a Weibull representation. Al_2O_3, $\sigma = 185$ MPa.

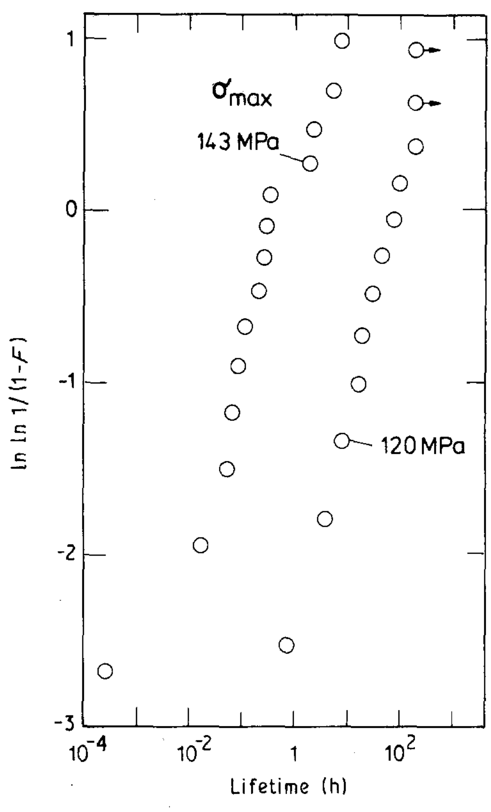

Figure 6 Cyclic lifetimes in a Weibull representation.

The Weibull plot for two stress amplitudes is shown in Fig. 6. The Weibull-parameters for $F < 0.72$ (corresponding to the strength) are

$$\sigma_{max} = 143\ \text{MPa}:$$

$$m^* = 0.654\ [0.41, 0.856],$$

$$t_0 = 0.62\ \text{h} \quad (N_0 = 1.1 \times 10^5)$$

$$\sigma_{max} = 120\ \text{MPa}:$$

$$m^* = 0.653\ [0.388, 0.868],$$

$$t_0 = 70\ \text{h} \quad (N_0 = 1.26 \times 10^7)$$

where the numbers in brackets are the lower and upper values of the 90% confidence interval.

6.2. Determination of da/dN–ΔK data

6.2.1. *Parameters A and n of a power-law description*

In order to determine the parameters A, A^* and n of the power law Equation 3, a least-squares fit according to Equation 9 was performed with the data of Fig. 5, resulting in

$$\log t_f = 60.99 - 28.64 \log \sigma_{max} \qquad (40)$$

or

$$\log N_f = 66.25 - 28.64 \log \sigma_{max} \quad (41)$$

with σ_{max} in MPa, t_f in h. From Equation 9 it follows that

$$\log N_f = \log\left(\frac{C}{2^n}\right) - n \log \sigma_{max} \quad (42)$$

Comparing this equation with the fitted straight line yields $n = 28.64$ and $\log C = 74.87$, and by use of the median strength, σ_{2c}, one obtains $\log B = 12.788$. As a consequence of Equation 25 and knowledge of the fracture toughness $K_{Ic} = 3.3$ MPa m$^{1/2}$, the factors A, A^* are given by $\log A = -27.94$, $\log A^* = -13.09$.

6.2.2. *Statistical procedure*

The statistical procedure for evaluating da/dN is relatively simple and will be outlined below for the natural cracks in the special load case of alternating bending ($R = -1$). The cyclic lifetime results of Fig. 5 are plotted once more in Fig. 6 as Weibull distributions. First, the cyclic fatigue data (N_f, t_f) have to be associated with the corresponding inert strength data, σ_c.

In the case of static tests, where the surface of failure (namely the surface in the tensile region) is predetermined, the lifetime, t_{fc}, and strength data are directly correlated.

$$F(t_f) = F(\sigma_c) \quad (43)$$

In the case of cyclic tests the correlation between strength and lifetime data is different. Because in alternating bending tests with natural cracks, the effective surface (volume) is twice the effective surface (volume), the cyclic lifetimes (or numbers to failure) have to be correlated to the strength data, σ_{c2}.

$$F(N_f) = F(\sigma_{c2}) \quad (44)$$

The auxiliary diagram, Fig. 7, provides the derivative necessary in Equation 32.

For application of the statistical procedure it is very important that lifetimes as well as strength measurements are performed using the same loading configuration. This is indispensable, because the scatter in lifetime and strength must not be influenced by use of different testing devices. From the data of Fig. 7 and application of Equation 32 the da/dN–ΔK data of Fig. 8a result. The relation can be described by

$$\frac{da}{dN} = 1 \times 10^{-13}\left(\frac{\Delta K}{K_{Ic}}\right)^{28.2} \text{ m/cycle} \quad (45)$$

with $K_{Ic} = 3.3$ MPa m$^{1/2}$.

In Fig. 8b the da/dt data for natural cracks [7] are represented. The data points can be described by Equation 39.

7. Results from specimens with Knoop cracks

A second series of specimens were damaged by Knoop indentation tests. Owing to the low strength of the 99.6% Al_2O_3 specimens with "natural" crack population, larger Knoop cracks had to be introduced to ensure fracture at these artificial cracks. This was

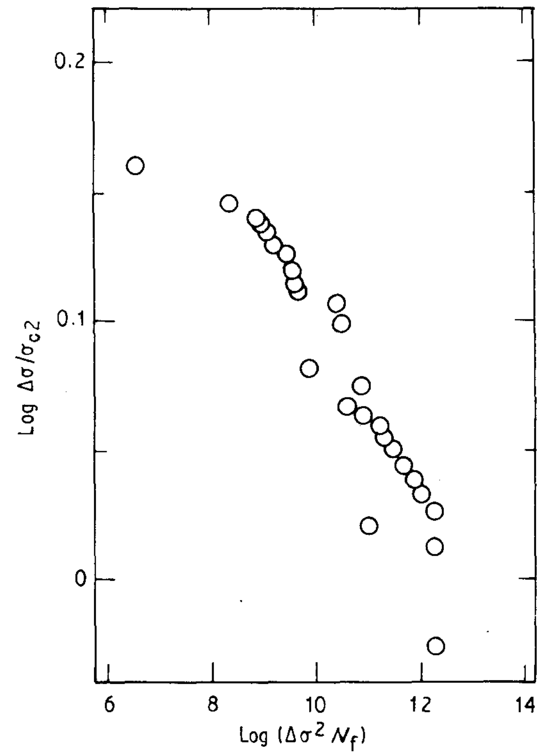

Figure 7 Auxiliary diagram for evaluating Equation 11 (natural cracks).

fulfilled for Knoop cracks obtained with an indentation load of 294 N. After indentation the Knoop-damaged specimens were annealed once more under the same annealing conditions in order to remove the residual stresses due to the indentations. Strength measurements yield a median strength of $\hat{\sigma}_c = 192$ MPa. Static lifetimes are represented in Fig. 9. A least-squares fit of the data yields an exponent $n_s = 38$. The related crack growth law is plotted in Fig. 8b as a solid line.

Fig. 10 shows the result of lifetime for a frequency of 50 Hz compared with predictions based on static tests. In this case, the prediction was made by use of Equation 21. Similar to the results for natural cracks, for the Knoop cracks also a significant cyclic fatigue effect is obvious.

7.1. Determination of da/dN–ΔK data

7.1.1. *Parameters A and n of a power-law description*

The same procedure as outlined for the natural cracks gives the parameters A, A^* and n of the power law Equation 3 for the artificial cracks by a least-squares fit of the data represented in Fig. 10

$$\log t_f = 43.76 - 21.05 \log \sigma_{max} \quad (46)$$

or

$$\log N_f = 49.02 - 21.05 \log \sigma_{max} \quad (47)$$

with σ_{max} in MPa, t_f in h. From this fit one obtains, with the median strength of $\sigma_c = 192$ MPa, for the Knoop cracks $n = 21.05$, $\log A = -22.93$, $\log A^* = -12.02$.

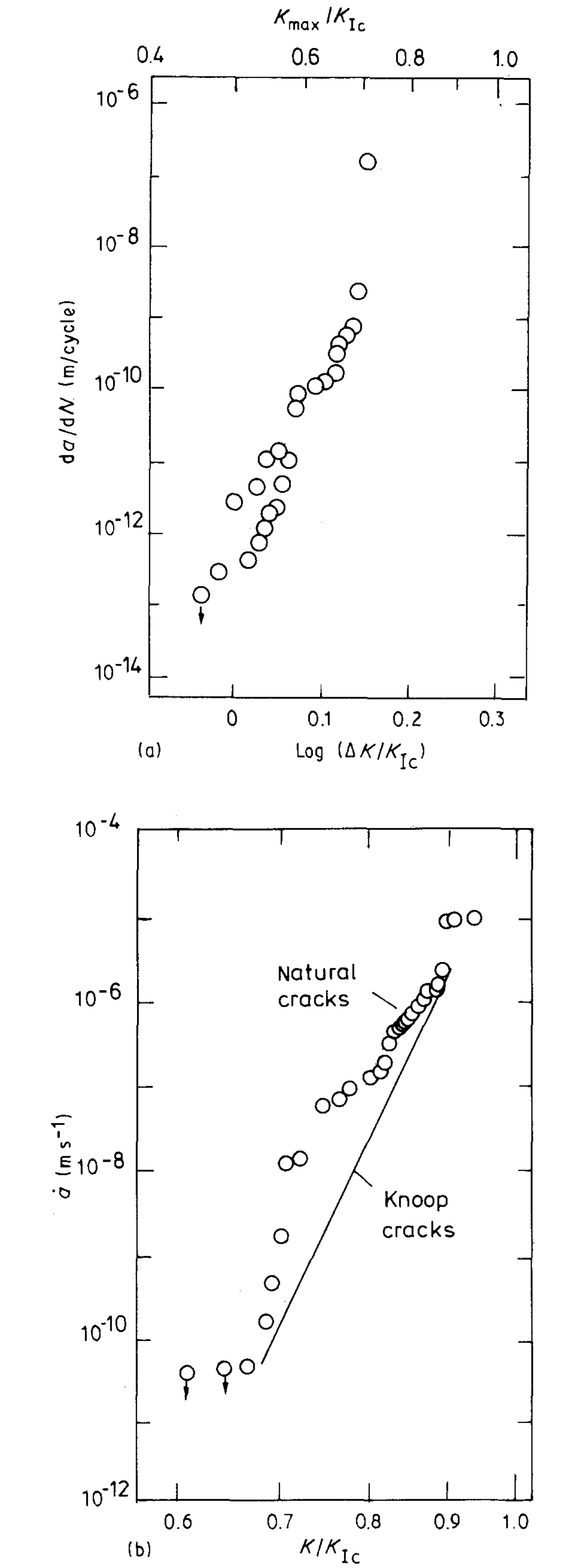

Figure 8 Crack growth curves for natural cracks. (a) da/dN–ΔK curve for cyclic tests at $R = -1$; (b) da/dt–K curve for static tests.

7.1.2. Least-squares procedure

In order to determine the fatigue crack growth law for Knoop cracks, the least-squares procedure is applied to the Knoop data represented in Fig. 10. In Fig. 11 the quantity $\log \Delta\sigma/\sigma_c$ is plotted versus $\log \Delta\sigma^2 N_f$. A least-squares fit by a quadratic polynomial yields

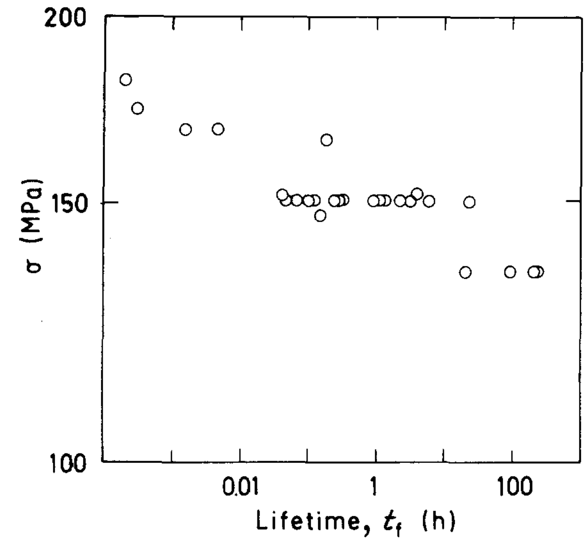

Figure 9 Static lifetimes for specimens with Knoop cracks.

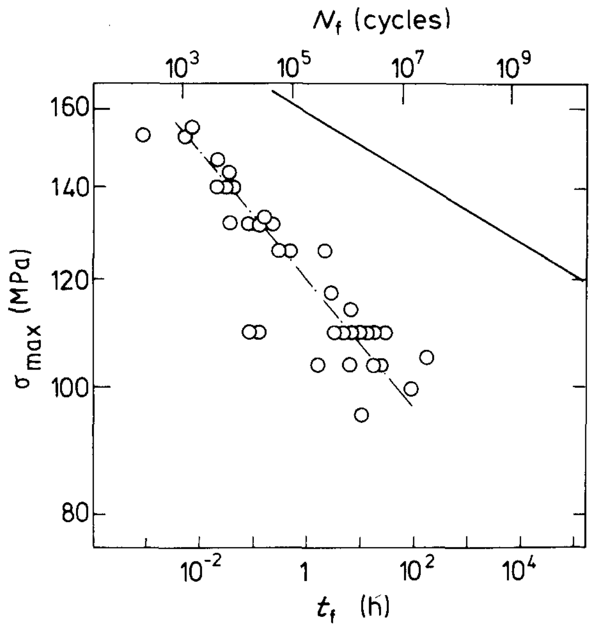

Figure 10 Cyclic lifetimes for specimens with Knoop cracks compared with predictions (——) based on static tests. R = − 1; 50 Hz.

$$\log(\Delta\sigma^2 N_f) = 12.21 - 20.745 \log \left(\frac{\Delta\sigma}{\sigma_c}\right) - 2.500 \log^2 \left(\frac{\Delta\sigma}{\sigma_c}\right) \quad (48)$$

Inserting these results in Equation 11a yields the da/dN–ΔK curves represented in Fig. 12. The dependencies can be approximated by a power law

$$\frac{da}{dN} = 3.9 \times 10^{-13} \left(\frac{\Delta K}{K_{Ic}}\right)^{23.2} \text{m/cycle} \quad (49)$$

In contrast to this result for crack growth under cyclic load, static lifetime measurements (Fig. 10) exhibited a subcritical crack growth exponent of $n_{static} \simeq 39$.

8. Discussion

From the results a pronounced cyclic fatigue effect can be concluded. The relation between crack growth rate

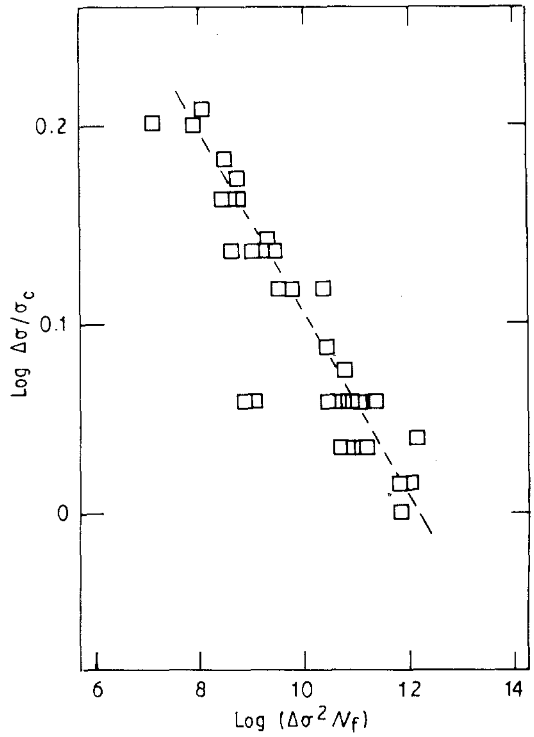

Figure 11 Auxiliary diagram for the Al_2O_3-ceramic with Knoop cracks.

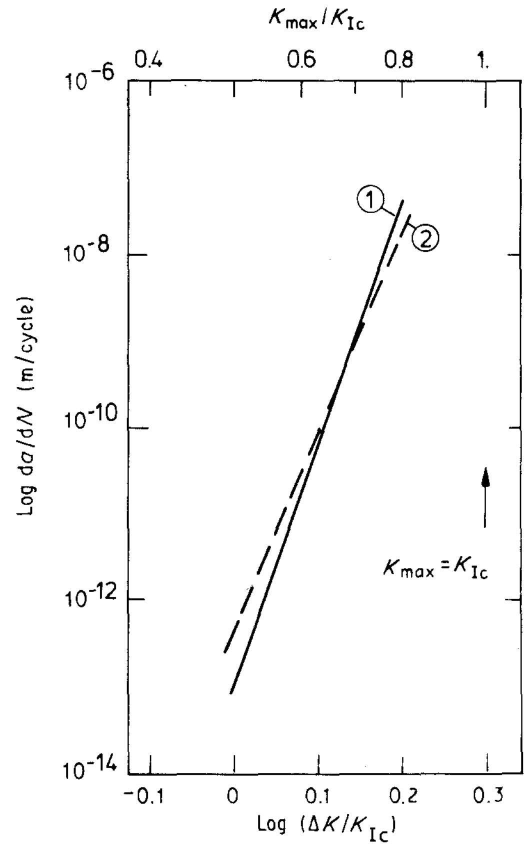

Figure 12 da/dN–ΔK curves for the two crack types: 1, natural cracks; 2, Knoop-cracks.

da/dN and ΔK can be described by a power law. The two methods applied to determine the parameters A^* and n yielded nearly identical values (see Table I).

The exponent is less than the exponent for sub-critical crack extension under static loading n_s.

TABLE I

Crack type, loading	A^*	n	Method
Natural cracks, cyclic	8.13×10^{-14}	28.6	Power law
	1.0×10^{-13}	28.2	Statistic
		34.3 (26, 53)	Equation 13a $\sigma_{max} = 143$ MPa
		34.2 (26, 56)	Equation 13a $\sigma_{max} = 120$ MPa
Knoop-cracks, cyclic	9.54×10^{-13}	21.1	Power law
	3.9×10^{-13}	23.2	Least squares
Natural cracks, static		38.6	Statistic
		32.5	Equation 13a

For the tests with the Knoop cracks the parameters are slightly – not significantly – different. From the good agreement between the different evaluation procedures it can be concluded that the assumption for the statistical procedure – namely the same flaw population for strength and cyclic fatigue – is fulfilled.

Another check of this assumption is the comparison of the Weibull parameters. The exponent n can be obtained from the Weibull parameters m of the strength and m^* of lifetime using Equation 13a. The deviation from the straight line in the Weibull-plot complicates this evaluation. Therefore, only the data for $F(\sigma_c) < 0.6$ and $F(t_s) < 0.6$ and for $F(\sigma_{2c}) < 0.72$ and $F(t_c, N_f) < 0.72$ are used. The results are shown in Table I.

For static loading the agreement is good ($n_s = 38.6$ from the evaluation of the lifetime, $n_s = 32.5$ from Equation 13a). For cyclic loading, Equation 13a leads to slightly higher values of n than the other method. Taking into account the limit values of the 90% confidence interval of the m^*-values, the limit values in brackets (Table I) are obtained, which include the n-values of 28.6 and 28.2 for the natural cracks. From this point of view there is no significant deviation between the various methods. It should, however, be emphasized that the determination of m^* may not be accurate enough because of the limited number of specimens tested.

9. Conclusion

Measurements of cyclic fatigue were carried out on an 99.6% Al_2O_3-ceramic in bending with an R-ratio $R = -1$. Two types of relatively small cracks have been considered: natural cracks and artificial cracks (Knoop cracks). By lifetime predictions based on static tests it has been proved that there is a real fatigue effect for both crack types.

In order to determine da/dN–ΔK curves for small cracks from lifetimes under cyclic load, a procedure has been derived which is based on a method usually applied to subcritical crack growth [4].

The main results are as follows.

1. Within the scatter of data, both crack types (natural and Knoop cracks) exhibit the same da/dN–ΔK relation.

2. The exponent of the Paris law ($n \simeq 23$–27) is significantly different from the exponent of the power law for subcritical crack growth $n \simeq 38$.

3. A threshold value ΔK_0 for fatigue crack growth is not detectable down to crack growth rates of $da/dN \simeq 1 \times 10^{-13}$ m/cycle.

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